

# Entropy of extremal warped black holes

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## Abstract

We study the entropy of extremal warped black hole obtained from the topologically massive gravity with a negative cosmological constant of  $\Lambda = -1/l^2$ . We compare the entropy  $S_e = \pi\alpha/3G$  from the Wald formalism with  $S_w = \pi lu/3G$  from the entropy function approach. These are the same if  $\alpha = lu$ . Also we obtain the same Cardy formula when  $J_e = l^3q$  with  $J_e$  the angular momentum and  $q$  the conserved quantity.

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# 1 Introduction

The gravitational Chern-Simons terms with  $K$  the gravitational Chern-Simons coupling constant produces a physically propagating massive graviton in three dimensional Einstein gravity [1]. This topologically massive gravity with a negative cosmological constant  $\Lambda = -1/l^2$  (TMG $_{\Lambda}$ ) gives us the BTZ black hole solution [2, 3]. For  $l/K > 3(\nu > 1)$ , there exists warped black hole solutions which are asymptotic to warped AdS $_3$  spacetimes [4]. These warped black holes are considered discrete quotients by an element of  $SL(2, R) \times U(1)$  of warped AdS $_3$ , as the BTZ black holes are discrete quotients of AdS $_3$ . Although thermodynamic quantities including the entropy have been investigated, their forms are very complicated. Especially, the entropy is still not fully understood [5, 6].

We note that the gravitational Chern-Simons terms are not invariant under coordinate transforms even they are conformally invariant [7, 8]. Thus, their variation of Cotton tensor plays no role in finding a new solution. All solutions of Einstein gravity are solutions of the TMG $_{\Lambda}$ . Hence, one needs to seek another way to investigate the TMG $_{\Lambda}$ . In this end, one may introduce conformal transformation to single out a conformal degree of freedom (dilaton). Then, the Kaluza-Klein ansatz is used to obtain an effective two-dimensional action of 2DTMG $_{\Lambda}$ , which will be a gauge and coordinate invariant. Saboo and Sen [9] have used the 2DTMG $_{\Lambda}$  to obtain the entropy of extremal BTZ black hole by using the entropy function formalism. This is possible because AdS $_2$  is stable attractor solution of equations which govern how the geometry changes as the degenerate horizon is approached. On the other hand, for a constant dilaton, the authors in [10, 11, 12] have employed the entropy function approach to find three distinct vacuum solutions of the 2DTMG $_{\Lambda}$ : AdS $_2$  with positive charge, AdS $_2$  with negative charge, and warped AdS $_2$  with positive charge. Upon uplifting to three dimensions, these were geometric solutions which are either AdS $_3$  or warped AdS $_3$  with an identification.

In this work, we explore the connection between two entropies of the extremal warped black holes obtained using the Wald formalism and the entropy function method.

## 2 Topologically massive gravity in Schwarzschild coordinates

The action for topologically massive gravity with a negative cosmological constant (TMG<sub>Λ</sub>) is given by [1, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22]

$$I_{\text{TMG}_\Lambda} = \frac{1}{16\pi G_3} \int d^3x \sqrt{-g} \left[ R_3 - 2\Lambda + \frac{K}{2} \varepsilon^{lmn} \Gamma_{lq}^p \left( \partial_m \Gamma_{np}^q + \frac{2}{3} \Gamma_{mr}^q \Gamma_{np}^r \right) \right], \quad (1)$$

where  $\varepsilon$  is the tensor defined by  $\varepsilon/\sqrt{-g}$  with  $\varepsilon^{012} = 1$ . We choose the Newton's constant  $G_3 > 0$ . The Latin indices of  $l, m, n, \dots$  denote three dimensional tensors. The  $K(=1/\mu)$ -term is called the gravitational Chern-Simons terms. Here we choose “+” sign to avoid negative graviton energy [14]. It is the first higher derivative correction in three dimensions because it is the third-order derivative.

Varying this action leads to the Einstein equation

$$G_{mn} + K C_{mn} = 0, \quad (2)$$

where the Einstein tensor including the cosmological constant is given by

$$G_{mn} = R_{3mn} - \frac{R_3}{2} g_{mn} - \frac{1}{l^2} g_{mn} \quad (3)$$

and the Cotton tensor is

$$C_{mn} = \varepsilon_m{}^{pq} \nabla_p \left( R_{3qn} - \frac{1}{4} g_{qn} R_3 \right). \quad (4)$$

We note that the Cotton tensor  $C_{mn}$  vanishes for any solution to Einstein gravity, so all solutions to general relativity are also solutions of the TMG<sub>Λ</sub>. Hence, for  $K = 0$ , the BTZ black hole solution [23] is obtained as the vacuum solution to the Einstein gravity with  $K = 0$  ( $G_{mn} = 0$ ),

$$ds_{BTZ}^2 = -N^2(r) dt^2 + \frac{dr^2}{N^2(r)} + r^2 \left( d\theta + N^\theta(r) dt \right)^2, \quad (5)$$

where the metric function  $N^2(r)$  and the lapse function  $N^\theta(r)$  are given by

$$N^2(r) = -8G_3 m + \frac{r^2}{l^2} + \frac{16G_3^2 j^2}{r^2}, \quad N^\theta(r) = -\frac{4G_3 j}{r^2}. \quad (6)$$

Here  $m$  and  $j$  are the mass and angular momentum of the BTZ black hole, respectively.

On the other hand, for  $\nu^2 = (l/3K)^2 > 1$ , the warped black hole solution is obtained as the vacuum solution to the TMG<sub>Λ</sub> [4, 24, 5, 25, 6, 26, 27]

$$ds_{wBH}^2 = -\tilde{N}^2 dt^2 + \frac{l^4}{4\tilde{R}^2 \tilde{N}^2} dr^2 + l^2 \tilde{R}^2 \left( d\theta + \tilde{N}^\theta dt \right)^2, \quad (7)$$

where

$$\begin{aligned}\tilde{R}^2(r) &= \frac{r}{4} \left( 3(\nu^2 - 1)r + (\nu^2 + 3)(r_+ + r_-) - 4\nu\sqrt{r_+r_-(\nu^2 + 3)} \right), \\ \tilde{N}^2(r) &= \frac{l^2(\nu^2 + 3)(r - r_+)(r - r_-)}{4\tilde{R}^2(r)}, \\ \tilde{N}^\theta(r) &= \frac{2\nu r - \sqrt{r_+r_-(\nu^2 + 3)}}{2\tilde{R}^2(r)}.\end{aligned}\tag{8}$$

We note that this warped black hole reduces to the BTZ black hole in a rotating frame when choosing  $\nu^2 = 1$ . Using the surface integral expressions, thermodynamic quantities of the ADT mass  $M$  and angular momentum  $J$  are obtained as

$$M = \frac{(\nu^2 + 3)}{24G} \left( r_+ + r_- - \frac{1}{\nu} \sqrt{r_+r_-(\nu^2 + 3)} \right),\tag{9}$$

$$J = \frac{\nu l(\nu^2 + 3)}{96G} \left[ \left( r_+ + r_- - \frac{1}{\nu} \sqrt{r_+r_-(\nu^2 + 3)} \right)^2 - \frac{(5\nu^2 + 3)}{4\nu^2} (r_+ - r_-)^2 \right].\tag{10}$$

We note the length dimensions of  $M$  and  $J$ :  $[M] = 0$ ,  $[J] = 2$  with  $[G] = [l] = [K] = 1$ . It was conjectured that the warped black holes are holographically dual to a two-dimensional conformal field theory (CFT<sub>2</sub>). The study of thermodynamics of these black holes have provided a strong support on the AdS<sub>3</sub>/CFT<sub>2</sub> correspondence. Actually, the left sector of the CFT<sub>2</sub> is independent of the right sector. At the thermal equilibrium, the two sectors have different temperatures: right and left moving temperatures given by

$$T_R = \frac{(\nu^2 + 3)(r_+ - r_-)}{8\pi l},\tag{11}$$

$$T_L = \frac{(\nu^2 + 3)}{8\pi l} \left[ r_+ + r_- - \frac{1}{\nu} \sqrt{r_+r_-(\nu^2 + 3)} \right].\tag{12}$$

Using these, one defines the Hawking temperature  $T_H$

$$\frac{1}{T_H} = \frac{4\pi\nu l}{\nu^2 + 3} \frac{T_L + T_R}{T_R}.\tag{13}$$

The entropy is calculated by using the Wald method which is a prescription for handling higher derivative terms. It is composed of two terms from Einstein action and Chern-Simons action as

$$S = \frac{\pi}{24\nu G} \left[ (9\nu^2 + 3)r_+ - (\nu^2 + 3)r_- - 4\nu\sqrt{r_+r_-(\nu^2 + 3)} \right].\tag{14}$$

Defining right and left moving charges

$$c_R = \frac{(5\nu^2 + 3)l}{G\nu(\nu^2 + 3)},\tag{15}$$

$$c_L = \frac{4\nu l}{G(\nu^2 + 3)},\tag{16}$$

the above entropy can be rewritten as

$$S = \frac{\pi^2}{3} (c_L T_L + c_R T_R). \quad (17)$$

This is the formula for the entropy of CFT<sub>2</sub> with central charges  $c_L$  and  $c_R$  at temperatures  $T_L$  and  $T_R$ . Introducing right and left moving energies

$$E_R = \frac{\pi^2 l c_R T_R^2}{6}, \quad E_L = \frac{\pi^2 l c_L T_L^2}{6}, \quad (18)$$

one expresses the angular momentum in terms of a difference of these energies

$$J = l(E_L - E_R). \quad (19)$$

Here we observe the length dimensions of  $[E_R] = [E_L] = 1$ ,  $[T_R] = [T_L] = 0$ ,  $[c_R] = [c_L] = 0$ ,  $[S] = 0$ . Furthermore, one could express the entropy as sum of the square roots of left and right energies

$$S = 2\pi \left( \sqrt{\frac{E_R}{6l}} c_L + \sqrt{\frac{E_L}{6l}} c_R \right), \quad (20)$$

which seems to be a Cardy formula for CFT<sub>2</sub>. In this sense, the conserved charges  $(E_L, E_R)$  and potentials  $(T_L, T_R)$  are more natural for describing the warped black holes than  $(M, J)$  and  $(T_H, \Omega_H)$  with  $\Omega_H = 2/(\nu r_+ - \sqrt{(\nu^2 + 3)r_+ r_-})l$  the angular velocity of the horizon.

In this work, this formalism is attractive because we consider the extremal warped black hole with  $T_R = 0$  ( $E_R = 0$ ) at  $r_+ = r_- \equiv r_e$ . In this case, the relevant quantities are angular momentum and left moving temperature

$$J_e = \frac{\nu l (\nu^2 + 3) r_e^2}{96G} \left( 2 - \frac{\sqrt{\nu^2 + 3}}{\nu} \right)^2, \quad T_L^e = \frac{(\nu^2 + 3) r_e}{8\pi l} \left( 2 - \frac{\sqrt{\nu^2 + 3}}{\nu} \right). \quad (21)$$

Importantly, the entropy for extremal warped black hole takes the form

$$S_e = \frac{\pi \nu r_e}{6G} \left( 2 - \frac{\sqrt{\nu^2 + 3}}{\nu} \right). \quad (22)$$

Here we observe that the positive entropy is guaranteed only for  $\nu > 1$ . Also,  $S_e$  could be recovered from the relation of  $S_e = \pi^2 c_L T_L^e / 3$ . At this stage, since we do not know the location  $r_e$  of extremal black hole explicitly, we could eliminate  $r_e$  by rewriting  $S_e$  in terms of  $J_e$  and  $c_L$  as

$$S_e = 2\pi \sqrt{\frac{J_e}{6l^2}} c_L = 2\pi \sqrt{\frac{E_L^e}{6l}} c_L \quad (23)$$

which looks like the Cardy formula for left mover. Here we used a relation of  $J_e = lE_L^e$ , which shows the extremality of warped black hole. However, the relation between  $S_e$  and  $M_e$  is not what one really wants to find like the Cardy formula

$$S_e = \left( \frac{4\pi\nu}{\nu^2 + 3} \right) M_e, \quad (24)$$

with

$$M_e = \frac{(\nu^2 + 3)r_e}{24G} \left( 2 - \frac{\sqrt{\nu^2 + 3}}{\nu} \right). \quad (25)$$

It shows a linear relation between  $S_e$  and  $M_e$ . Another interesting relation is

$$M_e = \left[ \frac{\pi l}{3G} \right] T_L^e. \quad (26)$$

Finally, the first law of thermodynamics seems to be

$$d\left(\frac{E_L^e}{l}\right) = T_L^e dS_e. \quad (27)$$

We note that the above description of extremal warped black hole might not be the only type of geometry which should be considered as a black hole. The boundary of the Poincare patch of  $\text{AdS}_2$  or  $\text{AdS}_3$  is an event horizon when these spaces arise as near-horizon limits of extremal warped black holes. There is a Killing horizon associated to time translations but no singularity behind it. In the next section, we will study the extremal warped black hole using Poincare coordinates.

### 3 Entropy function approach in Poincare coordinates

We first make conformal transformation and then perform dimensional reduction using the metric [7, 8]

$$ds_{DR}^2 = \phi^2 \left[ g_{\mu\nu}(x) dx^\mu dx^\nu + (dy + A_\mu(x) dx^\mu)^2 \right], \quad (28)$$

where  $y$  is coordinate that parameterizes an  $S^1$  with period  $2\pi l$ . Hence, its isometry is factorized as  $\mathcal{G} \times U(1)$ . After “ $y$ ”-integration, the action (1) reduces to the effective two-dimensional action of 2DTMG $_\Lambda$  [10, 11, 12]

$$\begin{aligned} I_{2\text{DTMG}_\Lambda} &= \frac{l}{8G_3} \int d^2x \sqrt{-g} \left( \phi R + \frac{2}{\phi} g^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi + \frac{2}{l^2} \phi^3 - \frac{1}{4} \phi F_{\mu\nu} F^{\mu\nu} \right) \\ &+ \frac{Kl}{32G_3} \int d^2x (R \epsilon^{\mu\nu} F_{\mu\nu} + \epsilon^{\mu\nu} F_{\mu\rho} F^{\rho\sigma} F_{\sigma\nu}). \end{aligned} \quad (29)$$

Here  $R$  is the 2D Ricci scalar with  $R_{\mu\nu} = Rg_{\mu\nu}/2$  and  $\phi$  is the dilaton. Also  $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$  is the field strength for gravivector  $A_\mu$  with  $\epsilon^{01} = 1$ . The Greek indices of  $\mu, \nu, \rho, \dots$  represent two dimensional tensors. We note that this action was used to derive the entropy of extremal BTZ black hole when applying the entropy function approach [9, 10].

Introducing a dual notation of  $F$  through  $F^{\mu\nu} = \frac{1}{\sqrt{-g}}\epsilon^{\mu\nu}F$ , equations of motion for dilaton  $\phi$  and gravivector  $A_\mu$ , respectively, are given by

$$R + \frac{2}{\phi^2}(\nabla\phi)^2 - \frac{4}{\phi}\nabla^2\phi + \frac{6}{l^2}\phi^2 + \frac{1}{4}F^2 = 0, \quad (30)$$

$$\epsilon^{\mu\nu}\partial_\nu \left[ \phi F - \frac{K}{2}(R + 3F^2) \right] = 0. \quad (31)$$

The trace part of Einstein equation

$$\nabla^2\phi - \frac{2}{l^2}\phi^3 + \frac{1}{2}\phi F^2 - K \left( \frac{1}{2}RF + F^3 + \frac{1}{2}\nabla^2F \right) = 0 \quad (32)$$

is relevant to the entropy calculation. Eq.(31) implies

$$\phi F - \frac{K}{2}(R + 3F^2) = \mathcal{C} \quad (33)$$

where  $\mathcal{C}$  is a constant related to a conserved quantity. Now, we wish to find  $\text{AdS}_2$  as the vacuum solution to Eqs. (30) and (32). In the case of a constant dilaton, we obtain the condition from these equations,

$$(3KF - 2\phi) \left( \frac{\phi^2}{l^2} - \frac{1}{4}F^2 \right) = 0, \quad (34)$$

which implies three relations between  $\phi$  and  $F$

$$\phi_\pm = \pm \frac{l}{2}F, \quad (35)$$

$$\phi_w = \frac{3K}{2}F. \quad (36)$$

$\phi_\pm = \pm lF/2$  denote the vacuum solutions to the 3D Einstein gravity. Hereafter we consider the case of (36) only because we are interested in the vacuum solution to the 2DTMG $_\Lambda$ . We note that for the case of  $K = l/3$ ,  $\phi_w = \phi_+$  which is a degenerate vacuum.

Assuming the line element preserving  $\mathcal{G} = SL(2, R)$  isometry

$$ds_{\text{AdS}_2}^2 = v \left( -x^2 d\psi^2 + \frac{dx^2}{x^2} \right), \quad (37)$$

we have the  $\text{AdS}_2$ -spacetimes, which satisfies

$$\bar{R} = -\frac{2}{v}, \quad \bar{\phi} = u, \quad \bar{F} = e/v \quad (\bar{F}_{10} = e). \quad (38)$$

In order to find an explicit form of the solution, we have to use the entropy function formalism because it provides an efficient way to find warped AdS<sub>2</sub>-solution as well as entropy of extremal warped black hole. The entropy function is defined as

$$\mathcal{E} = 2\pi \left( qe - \tilde{f}(e, v, u) \right) \quad (39)$$

where  $\tilde{f}(e, v, u)$  is the Lagrangian density  $\mathcal{L}_{2\text{DTMG}_A}$  evaluated when using Eq. (38),

$$\tilde{f} = \frac{l}{8G} \left[ -2u + \frac{2u^3v}{l^2} + \frac{ue^2}{2v} + \frac{K}{2} \left( \frac{2e}{v} - \frac{e^3}{v^2} \right) \right]. \quad (40)$$

Here we have equations of motion upon the variation of  $\mathcal{E}$  with respect to  $u$ ,  $v$ , and  $e$

$$-2 + \frac{6u^2v}{l^2} + \frac{e^2}{2v} = 0, \quad (41)$$

$$\frac{2u^3}{l^2} - \frac{ue^2}{2v^2} - K \left( \frac{e}{v^2} - \frac{e^3}{v^3} \right) = 0, \quad (42)$$

$$q = \frac{l}{8G} \left[ \frac{ue}{v} - \frac{K}{2} \left( \frac{2}{v} - \frac{3e^2}{v^2} \right) \right]. \quad (43)$$

For a consistency check, we mention the length dimensions of  $[v] = 2$ ,  $[u] = 0$ ,  $[e] = 1$ ,  $[q] = -1$ . Equations (41) and (42) are those obtained by plugging Eq. (38) into Eqs. (30) and (32). This means that the entropy function formalism uses the Einstein equation and dilaton equation in near-horizon geometry AdS<sub>2</sub>  $\times$   $S^1$  of the extremal warped black hole. The difference is that Eq. (31) is trivially satisfied with Eq. (38), while Eq. (43) is useful for deriving the entropy by choosing  $q = \frac{l\mathcal{C}}{8G}$ . The conserved quantity  $q$  measures momentum along  $y$ , which for warped black hole, is closely related to the angular momentum  $J_e$  [9].

Here we obtain the warped AdS<sub>2</sub>-solution for  $u = 3Ke/2v$  ( $\phi_w = 3FK/2$ ),

$$u = \sqrt{\frac{72GKql}{l^2 + 27K^2}}, \quad v = \frac{Kl}{8Gq}, \quad e = \sqrt{\frac{Kl^3}{2Gq(l^2 + 27K^2)}}, \quad q > 0. \quad (44)$$

Then, plugging these into Eq.(39) leads to the entropy of the extremal warped black hole

$$S_w = \frac{2\pi}{eG} \frac{Kl^3}{l^2 + 27K^2} = 4\pi eq \simeq 2\pi \sqrt{\frac{ql}{6}} c_L, \quad K > 0, \quad (45)$$

where the left moving central charge (16) takes the form

$$c_L = \frac{24qe^2}{l} \quad (46)$$



when employing the twisted energy momentum tensor for the  $k = 8ql^2$  level of  $U(1)$  current with  $c_L = 3ke^2/l^4$  [10]. We check the correct length dimension  $[S_w] = 0$  of the entropy together with  $[ql] = 0$  and  $[c_L] = 0$ . The last relation ( $\simeq$ ) will be confirmed from the Cardy formula if  $ql$  is the eigenvalue of  $L_0$ -operator of dual CFT<sub>2</sub>. If one assumes the relation

$$J_e = l^3 q, \quad (47)$$

one recovers the Cardy formula (23). We note that the background metric (28) can be rewritten as the extremal warped black hole in Poincare coordinates  $(\psi, x, z)$  [4]

$$ds_{DR}^2 = (\bar{\phi})^2 \left[ \bar{g}_{\mu\nu} dx^\mu dx^\nu + (dy + \bar{A}_\mu dx^\mu)^2 \right] \quad (48)$$

$$= u^2 v \left( -x^2 d\psi^2 + \frac{dx^2}{x^2} \right) + (ue)^2 (dz^2 + x d\psi)^2 \quad (49)$$

$$= \frac{l^2}{\nu^2 + 3} \left[ -x^2 d\psi^2 + \frac{dx^2}{x^2} + \frac{4\nu^2}{\nu^2 + 3} (dz + x d\psi)^2 \right] \quad (50)$$

with  $y = ez$ . This appears to be independent of  $q$ .

Finally, we express the extremal entropy  $S_w$  in Eq.(45) in terms of Poincare coordinates as

$$S_w = \frac{\pi l u}{3G}, \quad u = \bar{\phi} = \sqrt{\frac{24G\nu q}{\nu^2 + 3}}. \quad (51)$$

In Schwarzschild coordinates, self-dual solutions are obtained by identifying time  $t$ , i.e.  $t \sim t + 2\pi\alpha$ . The left moving temperature is given by  $T_L = (\nu^2 + 3)\alpha/(4\pi\nu l)$ . Then, the entropy (22) takes the form

$$S_e = \frac{\pi\alpha}{3G} \quad (52)$$

with

$$\alpha = \frac{\nu r_e}{2} \left( 2 - \frac{\sqrt{\nu^2 + 3}}{\nu} \right). \quad (53)$$

Since the entropy is an invariant quantity, comparing  $S_w$  with  $S_e$  leads to the important relation

$$\alpha = l u. \quad (54)$$

We may understand the relation between  $S_e$  and  $S_w$  by considering the coordinate transformations

$$t = -\frac{2\alpha}{\nu^2 + 3}\psi + \frac{2\nu}{\nu^2 + 3}z, \quad (55)$$

$$\theta = \frac{2}{\nu^2 + 3}\psi, \quad (56)$$

$$r = x + r_e. \quad (57)$$

The location  $r = r_e$  of extremal warped black hole appears at the boundary of  $x = 0$ . We observe “ $\alpha$ ” in Eq.(55), even though there does not exist the exact relation of Eq.(54).

In conclusion, in order to have the same entropy for extremal warped black hole, we observe the correspondences between two approaches as

$$\alpha \longleftrightarrow l u, \quad J_e \longleftrightarrow l^3 q, \quad (58)$$

where the latter is introduced for obtaining the same Cardy formula. The difference is that in the Wald formalism with Schwarzschild coordinates, the positive entropy is allowed for  $\nu > 1$  and the extremal entropy is problematic at  $\nu = 1$ , while in the entropy function formalism with Poincare coordinates, the positive entropy is guaranteed for  $\nu > 0$  and the extremal entropy is properly obtained for  $\nu = 1$ .

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